3/21/17

"The question isn't 'Who is going to let me?'; It's 'Who is going to stop me?'." -Ayn Rand

HW: "Logarithmic Functions" homework section
Test 3 on Thursday 3/30

AIM: What is a Logarithmic Function?

Warm Up: (Do it on the back of your packet)

1) If the graph of the exponential function passes through the points (0,9) and (4, 16/9), what is the equation of the exponential function?

y-int $\therefore$ $a=9$

$y = ab^x$

use $(4, \frac{16}{9})$

$$\frac{\frac{16}{9}}{9} = \frac{9b^4}{9}$$

$$\frac{16}{81} = b^4$$

$$b = \frac{2}{3}$$

Eq:
$y = 9\left(\frac{2}{3}\right)^x$

Alt: $(0,9)$ $(4, \frac{16}{9})$

$$\frac{16}{9} = ab^4$$

$$9 = ab^0$$

$$\frac{\frac{16}{9}}{9} = \frac{\cancel{a}b^4}{\cancel{a}\cancel{b^0}}$$

$$\frac{16}{81} = b^4$$

$$b = \frac{2}{3}$$

$$9 = a\left(\frac{2}{3}\right)^0$$

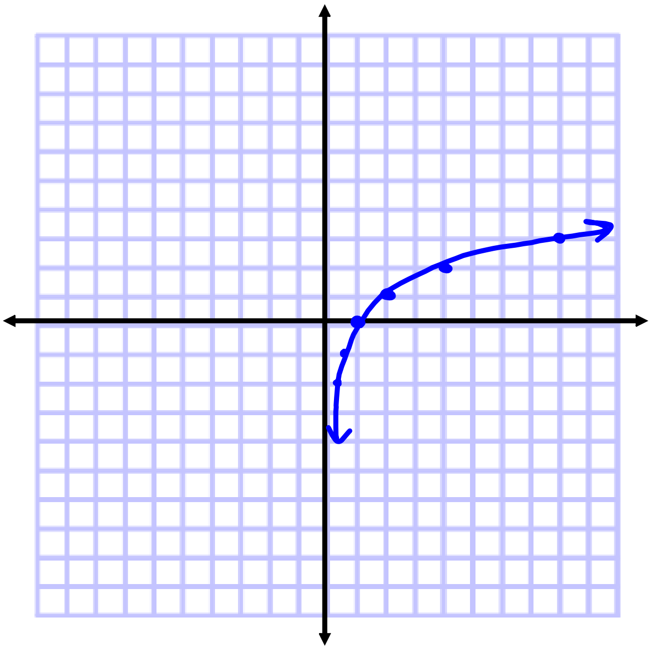
$$9 = a(1)$$

$$a = 9$$

One of the properties of the exponential function $f(x) = b^x$ is that it is a 1-1 function. Remember that this means it has an inverse function whose graph can be obtained by reflecting the graph of $y = b^x$ through the line $y = x$.

switch x and y

Let's graph the inverse of the function that we were looking at yesterday, $y = 2^x$.



Inverse

x	y
1/4	-2
1/2	-1
1	0
2	1
4	2
8	3

x	y
-2	1/4
-1	1/2
0	1
1	2
2	4
3	8

Write an equation for the inverse of $y = 2^x$.

switch x and y

$$\boxed{x = 2^y}$$

log means exponent

The equation $x = b^y$ tells us that y is the exponent on b that produces x. In situations like this the word logarithm is used in place of exponent. A **logarithm** is an exponent. We can abbreviate to:

$$y = \log_b x$$

"y is the exponent of b that gives us x as an answer."

read as "y equals log x to the base b" or "y equals log x base b."

answer base exponent

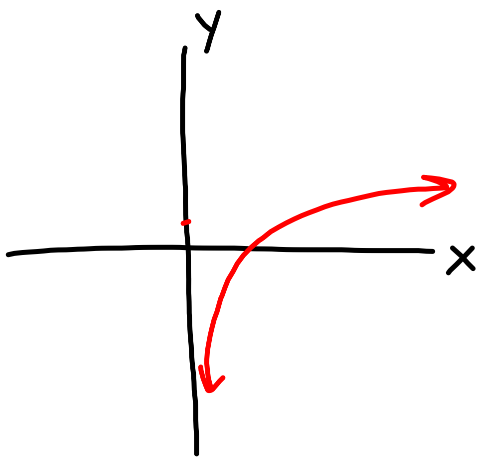
Exponential form: $x = b^y$

Logarithmic form: $y = \log_b x$

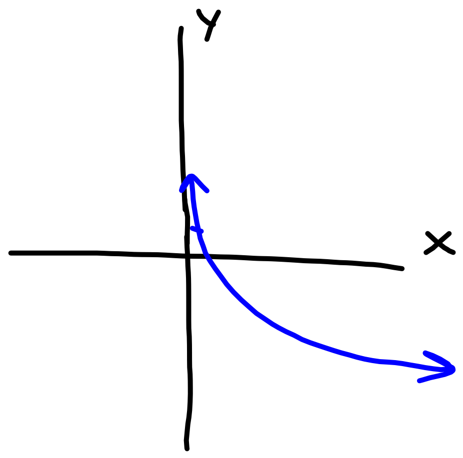
means the same thing

exponent base answer

General Graphs of Logarithmic Functions



$y = \log_b x, b > 1$



$y = \log_b x, 0 < b < 1$

Properties of Logarithmic Functions

1. The domain consists of all positive numbers.
2. The range consists of all real numbers.
3. The function is increasing (the curve is rising) when $b > 1$, and its is decreasing (the curve is falling) when $0 < b < 1$.
4. It is a one-to-one function.
5. The point (1,0) is on the curve.
6. There is no y-intercept.
7. The y-axis is a vertical asymptote to the curve.

Practice

1. Evaluate the following logarithms. If needed, write an equivalent exponential equation. (Without the use of your calculator.)

(a) $\log_2 8$

"exponent of 2 that gives us 8"

$2^x = 8$

$2^x = 2^3$

$\boxed{x=3}$

(b) $\log_4 16$

$4^x = 16$

$\boxed{x=2}$

(c) $\log_5 625$

$5^x = 625$

$\boxed{x=4}$

(d) $\log_{10} 100{,}000$

$10^x = 100000$

$x = 5$

(e) $\log_6 \left(\frac{1}{36}\right)$

$6^x = \frac{1}{36}$

$6^x = 36^{-1}$

$6^x = (6^2)^{-1}$

$6^x = 6^{-2}$

$x = -2$

(f) $\log_2 \left(\frac{1}{16}\right)$

$2^x = \frac{1}{16}$

$2^{-4} = \frac{1}{16}$

-4

(g) $\log_5 \sqrt{5}$

$5^x = \sqrt{5}$

$5^x = 5^{1/2}$

$\frac{1}{2}$

(h) $\log_3 \sqrt[5]{9}$

$3^x = \sqrt[5]{9}$

$3^x = 9^{1/5}$

$3^x = (3^2)^{1/5}$

$3^x = 3^{2/5}$

$x = \frac{2}{5}$

It is critically important to understand that logarithms **give exponents as their outputs**. We will be working for multiple lessons on logarithms and a basic understanding of their inputs and outputs is critical.

2. If the function $y = \log_2(x+8)+9$ was graphed in the coordinate plane, which of the following would represent its *y*-intercept?

$x = 0$

(1) 12 (3) 8

(2) 13 (4) 9

$y = \log_2(0+8)+9$

$y = \log_2(8)+9$

$y = 3+9 = 12$

3. Between which two consecutive integers must $\log_3 40$ lie?

(1) 1 and 2 (3) 3 and 4

(2) 2 and 3 (4) 4 and 5

$3^3 = 27$

$3^4 = 81$

Calculator Use and Logarithms – Most calculators only have two logarithms that they can evaluate directly. One of them, $\log_{10} x$, is so common that it is actually called the **common log** and typically is written without the base 10.

$$\log x = \log_{10} x \quad \text{(The Common Log)}$$

4. Evaluate each of the following using your calculator.

(a) $\log 100$

10 to what power is 100?

$\boxed{2}$

(b) $\log\left(\frac{1}{1000}\right)$

$10^x = \frac{1}{1000}$

$10^x = 1000^{-1}$

$10^x = (10^3)^{-1}$

$\boxed{x = -3}$

(c) $\log\sqrt{10}$

$10^x = \sqrt{10}$

$10^x = 10^{\frac{1}{2}}$

$\boxed{x = \frac{1}{2}}$

5. Can the value of $\log_2(-4)$ be found? What about the value of $\log_2 0$? Why or why not? What does this tell you about the domain of $\log_b x$?

There is no power that we can raise 2 to in order to get -4. There is also no power that we can raise 2 to in order to get 0. The domain must be x>0.